Steady flow through non-uniform gauzes of arbitrary shape

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The steady, two-dimensional flow through an arbitrarily-shaped gauze, of non-uniform properties, placed in a parallel channel is considered for the case in which viscosity can be ignored except in the immediate vicinity of the gauze. The equations are linearized by requiring departures from uniformity both in the flow and in the gauze parameters to be small. Knowledge of any three of the upstream profile, the downstream profile, the shape of the gauze, and the gauze parameters, allows the other to be calculated from a linear relation between these four quantities. Particular solutions are given for the production of a uniform shear and the flow through linear and parabolic gauzes. The validity of the solution is verified by experiment. It is shown that the method can also be applied to two-dimensional flow in a diverging channel, axisymmetric flow in a circular pipe and in a circular cone, and to flow through multiple gauzes.

1. Introduction

When a stream of fluid passes through a wire gauze the stream may be deflected and the static pressure of the stream reduced. An adequate description of these properties of the gauze can be given in terms of a drag coefficient, K, and a lift coefficient, B, and many, largely empirical attempts, have been made to relate these coefficients to the numerical parameters of the gauze (e.g. Wieghardt 1953).

The present paper is concerned principally with the steady two-dimensional flow in a parallel channel in which an arbitrarily-shaped gauze is placed. The flow through a gauze placed normal to the incident stream was first solved by Taylor & Batchelor (1949), who showed that departures from uniformity in the incident stream were attenuated by a factor $(2-K-B+KB)/(2+K-B)$. This solution was also obtained by Bonneville & Harper (1951), in a manner similar to that employed by Bragg & Hawthorne (1950), who considered the perturbation to the stream function by gauzes that deflected the streamlines by small amounts. Davis (1957) applied the method to flow through two normal, interfering gauzes, while Owen & Zienkiewicz (1957) showed that a gauze, with a linear variation of drag across its surface, placed normal to a uniform incident stream produced a linear profile downstream. All these investigations rely on the departures of the flow from uniformity being small. A similar perturbation solution will be obtained

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below for the flow through a single or a pair of arbitrarily shaped, non-uniform gauzes; the equations being linearized by requiring both the departures of the gauze parameters from uniformity and the angle of incidence of the flow on the gauze to be small. The principal result of the paper is a linear relation between the upstream velocity profile, the downstream profile, the shape of the gauze, and the variation of K across the gauze. This relation can be manipulated to give any one of these four quantities in terms of the other three and contains all the hitherto known solutions as particular cases.

Although in principle the solution also applies to a gauze which only partially fills the channel, it is found by experiment to be invalid unless $K \ll 1$. For higher values of K the only solution so far known is that given by Taylor (1944), which consists in replacing the gauze by a sheet of sources. Throughout this paper only gauzes which fill the channel will be considered.

1.1. *A gauze as a surface of discontinuity*

For the present purpose any regular or nearly regular spatial distribution of obstructions which lie on or near a single surface will be regarded as a gauze. Typical examples are: a row of cylinders, a plane rectangular mesh of circular wires, a perforated plate, a sheet of cloth, a cascade of aerofoils, a honeycomb of parallel tubes, etc. Combinations of more than one, and possibly different, gauzes can under certain conditions be regarded as a single gauze. The definition implies and requires that the extent of the gauze is large in the directions parallel and small in the direction normal to the surface. As the following treatment is largely two-dimensional, a gauze may be thought of as a row of equal cylinders, not necessarily lying in a plane nor necessarily equally spaced.

Although 'gauze' is quite a broad concept, investigations are usually restricted to gauzes of simple geometrical form which can be specified by a few numbers, such as the wire spacing l and the wire diameter d of a square mesh wire gauze. The flow will then be determined by a velocity scale U_1, the kinematic viscosity ν, l, and d, from which the dimensionless parameters $\beta = (1-d/l)^2$, the proportion of open area presented to the stream, and the Reynolds number $R = U_1 d/\nu$, may be constructed.

Consider now a gauze placed in a steady-flow field, and use the suffices 1 and 2 to indicate the regions upstream and downstream of the gauze. Some distance away from the gauze, the exact nature of the flow near it will be of little importance, particularly for $R \gg 1$. On this basis the gauze may be replaced by a surface of discontinuity across which discontinuities in the velocity and pressure fields occur. It remains to describe how the fields immediately on either side of the surface are related.

Let the velocity at the gauze be resolved into a normal component U_n, and a tangential component V_s (see figure 1). The mass flux ρU_n of fluid of density ρ through the gauze is conserved, and since the fluid is incompressible the velocity change

$$\Delta U_n \equiv U_{n1} - U_{n2} = 0.$$

Similarly, the momentum flux along the normal to the gauze is conserved, so that the drag per unit area is given solely by Δp, the loss of pressure across the gauze. It is convenient to express this in terms of a dimensionless resistance coefficient K defined by

$$\Delta p = \tfrac{1}{2}\rho K U_n^2. \tag{1.2}$$

K may take all values $\geqslant 0$. By Bernoulli's theorem it follows that Δp is also given by the change in total pressure, $p + \frac{1}{2}\rho U^2$, from far upstream to that far downstream along a streamline.

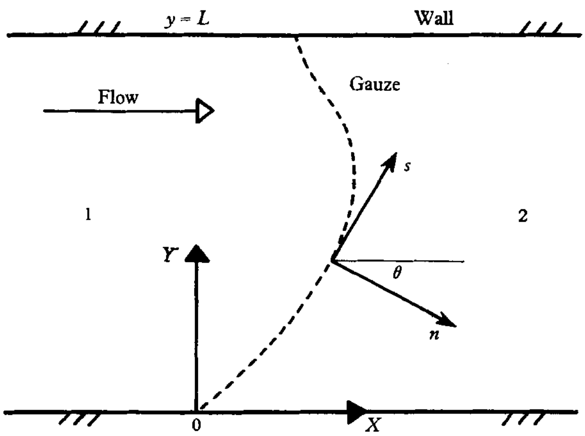

FIGURE 1. Diagram of the co-ordinate systems and the flow boundaries.

The screen also, in general, experiences a lift force, so that $V_{s1} \neq V_{s2}$, and we can define B such that

$$\Delta V_s \equiv V_{s1} - V_{s2} = BV_{s1}. \tag{1.3}$$

A streamline is usually bent towards the normal so that $0 \leqslant B \leqslant 1$. It may be that $B = B(\phi)$, where ϕ is the angle between V_{s1} and some characteristic direction in the gauze, such as a wire direction. However, it will be assumed in this paper, as hitherto, that B is independent of ϕ, and further that U_n, V_{s1} and V_{s2} lie in the same plane.

Davis (1957) has found by experiment that K varies with the Reynolds number R in the manner

$$K \doteqdot K_0 + 88(1-\beta)/R. \tag{1.4}$$

K_0 is independent of R and can be related to β by a mixing-jet model, similar to that proposed by Taylor & Davies (1944), with the result that

$$K_0 = [(1-0{\cdot}95\beta)/0{\cdot}95\beta]^2, \tag{1.5}$$

where $0{\cdot}95$ is an empirical constant.

The mechanism responsible for B is not yet understood. If, however, the flow is equivalent to a uniform stream past a row of equally spaced vortices of circulation k, then $V_1 = V_2 + 2\pi k/l$. Hence, by definition, $B = 2\pi k/lV_1$. If such a circulation exists it must be generated in the vicinity of the cylinder so that on dimensional grounds $k = AdV_1$, where A is a constant. That is, $B = 2\pi Ad/l$. For a square mesh wire gauze,

$$\beta = (1-d/l)^2 \quad \text{and} \quad K \doteqdot [(1-\beta)/\beta]^2$$

by (1.5), so that,

$$B = 1-(1+\sqrt{K})^{-\frac{1}{2}}, \tag{1.6}$$

where we have placed $2\pi A = 1$ to satisfy the assumption that $B = 1$ for $K = \infty$. This formula is similar to one proposed on empirical grounds by Taylor & Batchelor (1949), and although it may be considerably in error for a particular gauze, it is sufficiently realistic to provide a convenient choice of pairs of values of K, B for use in theoretical discussion. B will be assumed independent of both R and the angle of incidence of the flow.

K and B may, of course, be functions of position on the gauze, but there is no real significance in changes of K and B over distances much less than l.

1.2. *The linearized boundary conditions at the gauze*

Consider the flow through a gauze placed in a parallel channel with walls at $y = 0, L$ such that every velocity vector lies in the Cartesian co-ordinate plane XOY and such that the flow at infinity is parallel to OX and given by $U_\infty(y)$. Let the angle XOn be θ (see figure 1). If the effect of viscosity be neglected except in the vicinity of the gauze, vorticity is conserved along streamlines. If, further, the flow perturbation due to the presence of the gauze is sufficiently small so that streamlines are deflected by a small amount only, then the vorticity $\boldsymbol{\zeta} = \zeta\mathbf{k}$, where $\mathbf{k}$ is unit vector normal to XOY, is such that ζ is unchanged by the presence of the gauze except for a discontinuity at the gauze. Thus $\zeta = \zeta(y) = -\partial U_\infty/\partial y$.

It is necessary to establish that, in practice, it is possible to realize flows in which the assumption of a perfect fluid is valid. For the purpose of taking into account the diffusion of momentum by viscosity, consider as an extreme example the flow given by
$$U_{\infty 2} = V + U_0 \operatorname{erf}(Vy^2/4\nu x)^{\frac{1}{2}} \quad (U_0 \ll V).$$
This can be shown to correspond to the flow produced by the laminar mixing of two parallel streams of slightly different speeds, $V \pm U_0$, such as would be produced by a gauze made up of two parts of slightly different resistance coefficient. The half-width of the viscous region is required to be less than aL, say, where for practical purposes a lies between 0·01 and 0·05, and L is the width of the channel. Hence we require
$$x/L < 0{\cdot}28a^2VL/\nu.$$
This condition can normally be satisfied without difficulty. For example, in the Cavendish wind tunnel with $L = 40$ cm and $V = 500$ cm/sec, $x/L < 4$ even for $a = 0{\cdot}01$, and for $a = 0{\cdot}05$, $x/L < 100$. On the other hand, a uniform shear can be expected to persist almost indefinitely until viscous effects encroach into the flow from the wall boundary layers. Thus the assumption of a perfect fluid is a good one.

Write
$$U_1 = U, \quad T = \tan\theta, \quad \gamma = K\cos^2\theta, \tag{1.7}$$
and substitute into equations (1.1), (1.2) and (1.3), obtaining
$$U_2 = U - (V_1 - V_2)\,T,$$
$$\Delta p = \tfrac{1}{2}\rho\gamma(U^2 + V_1^2T^2),$$
$$BUT = (1-B)\,V_1 - V_2 + (V_1 - V_2)\,T^2.$$
Except at the gauze, the fluid obeys the equation of motion
$$-\frac{1}{\rho}\nabla p = \tfrac{1}{2}\nabla\mathbf{U}^2 - \mathbf{U}\times\boldsymbol{\zeta}, \quad \boldsymbol{\zeta} = \nabla\times\mathbf{U},$$

from the y-component of which

$$-\frac{1}{\rho}\frac{\partial}{\partial y}\Delta p = \frac{1}{2}\frac{\partial}{\partial y}\Delta \mathbf{U}^2 + (U_1\zeta_1 - U_2\zeta_2),$$

since $\zeta_1 = -\partial U_{\infty 1}/\partial y$, $\zeta_2 = -\partial U_{\infty 2}/\partial y$. The largest of the above terms involving T is $(V_1 - V_2)\,T$. For T near zero $(V_1 - V_2)\,T$ is small compared to U_1 so that neglecting such terms (a first step towards complete linearization) and making the equations dimensionless by means of a suitable mean velocity V by defining

$$q = U/V, \quad u = U_{\infty 1}/V, \quad u^* = U_{\infty 2}/V,$$
$$u_1 = U_1/V, \quad u_2 = U_2/V, \quad v_1 = V_1/V, \quad v_2 = V_2/V,$$

we have at the gauze,

$$\left.\begin{aligned} u_1 = u_2 &= q, && (a)\\ BqT &= (1-B)\,v_1 - v_2, && (b)\\ \frac{\partial}{\partial y}\gamma q^2 &= 2q(u' - u^{*\prime}) \quad (' = d/dy). && (c)\end{aligned}\right\} \tag{1.8}$$

Now $B \leqslant 1$ and BT is small over a range of θ near zero. If the departures of q from uniformity are also small, the product of BT and the variation of q can be neglected so that we can write $q = 1$ in BTq. The linearization is completed by writing

$$\gamma = \gamma_0[1 + s(y)], \tag{1.9}$$

where γ_0 is a constant and $|s| \ll 1$. Substituting (1.9) into (1.8c) and neglecting second-order quantities, we obtain after an immediate integration

$$u - u^* = \gamma_0(q-1) + \tfrac{1}{2}\gamma_0 s, \tag{1.10}$$

where to satisfy continuity we require $\int_0^L s(y)\,dy = 0$ if V is the mean velocity.

The derivation of (1.8) and (1.10) has required the following quantities to be small: the displacement of a streamline by the gauze, the variation of resistance s, $(V_1 - V_2)\,T$, BT, and the variation of velocity across the channel. It is found experimentally that it is normally sufficient to have the dimensionless shear $L\,du/dy$ or $L\,du^*/dy$ less than 0·5.

For low values of R, such that K behaves as in (1.4), equation (1.10) remains valid provided we write

$$\gamma = \cos^2\theta\, d(KV^2)/dV^2.$$

2. Two-dimensional flow in a parallel channel

The problem is to find the transformation relating the velocity profiles upstream and downstream of an arbitrary gauze. The fluid is assumed to be perfect, have zero-normal velocity on the channel walls and satisfy (1.8a, b) and (1.10) at the gauze. The velocity field will be formulated in terms of a stream function ψ which is perturbed in the vicinity of the gauze. The perturbation is determined by the assumption that vorticity is conserved along a streamline except for a discontinuity at the gauze.

The formulation is simplest for two-dimensional motion where (see, for example, Lamb 1932),

$$\zeta = \nabla^2\psi. \tag{2.1}$$

Suppose the gauze produces a perturbation ψ^* to a main flow ψ^0 such that $\psi = \psi^0 + \psi^*$, then, provided the gauze produces only small transverse displacements of the streamlines,

$$\nabla^2\psi^* = 0. \tag{2.2}$$

A finite solution of (2.2) which satisfies the boundary conditions at the walls is

$$\left.\begin{aligned}\frac{\psi^*}{LV} &= \sum_{n=1}^{\infty}\frac{1}{n\pi}P_n\, e^{n\pi x/L}\sin n\pi y/L \quad (x<0),\\ &= \sum_{n=1}^{\infty}\frac{1}{n\pi}Q_n\, e^{-n\pi x/L}\sin n\pi y/L \quad (x>0).\end{aligned}\right\} \tag{2.3}$$

Thus,

$$\left.\begin{aligned}u_1 &= u - \sum_1^{\infty} P_n\, e^{n\pi x/L}\cos n\pi y/L,\\ u_2 &= u - \sum_1^{\infty} Q_n\, e^{-n\pi x/L}\cos n\pi y/L,\\ v_1 &= \sum_1^{\infty} P_n\, e^{n\pi x/L}\sin n\pi y/L,\\ v_2 &= \sum_1^{\infty} Q_n\, e^{-n\pi x/L}\sin n\pi y/L.\end{aligned}\right\} \tag{2.4}$$

The velocities given by these equations must satisfy the boundary conditions (1.8) and (1.10) on the gauze. Assuming that the gauze is everywhere nearly coincident with the plane $x = 0$, these conditions are

$$q = u - \sum_1^{\infty} P_n \cos nw = u^* - \sum_1^{\infty} Q_n \cos nw, \tag{2.5}$$

$$BT = \sum_1 [(1-B)P_n + Q_n]\sin nw, \tag{2.6}$$

$$u - u^* = \gamma_0(q-1) + \tfrac{1}{2}\gamma_0 s, \tag{2.7}$$

where $w = \pi y/L$ and from (2.7) and (2.9),

$$\gamma_0(u-1) + \tfrac{1}{2}\gamma_0 s = \sum_1^{\infty}[(1+\gamma_0)P_n - Q_n]\cos nw. \tag{2.8}$$

It is convenient to define

$$\left.\begin{aligned}\alpha_n &= (1-B)P_n + Q_n,\\ \beta_n &= (1+\gamma_0)P_n - Q_n,\end{aligned}\right\} \tag{2.9}$$

so that,

$$\left.\begin{aligned}BT &= \sum_1^{\infty}\alpha_n \sin nw,\\ \tfrac{1}{2}\gamma_0 s + \gamma_0(u-1) &= \sum_1^{\infty}\beta_n \cos nw.\end{aligned}\right\} \tag{2.10}$$

Then

$$P_n = \frac{\alpha_n + \beta_n}{2+\gamma_0 - B}, \quad Q_n = \frac{-(1+\gamma_0)\alpha_n + (1-B)\beta_n}{2+\gamma_0-B}, \tag{2.11}$$

so that substituting q from (2.5) in (2.7),

$$u^* - 1 = A(u-1) + \tfrac{1}{2}(1-A)s + E\sum_1^\infty \alpha_n \cos nw, \tag{2.12}$$

where

$$E = \gamma_0(2+\gamma_0 - B) \quad \text{and} \quad A = (2-\gamma_0 - B + \gamma_0 B)/(2+\gamma_0 - B) = 1-\gamma_0(1-E).$$

Hardy & Rogosinski (1944) discuss the relation between two functions $g(w)$, $g^*(w)$ defined in the range $0 < w < \pi$ such that

$$g(w) = \sum_1^\infty h_n \sin nw \quad \text{and} \quad g^*(w) = \sum_1^\infty h_n \cos nw.$$

This relation is denoted by $g^* = \mathscr{H}(g)$ and $g = \mathscr{H}^*(g^*)$, and they show that

$$\mathscr{H}(g) = \frac{1}{2\pi}\int_0^\pi [g(w+t) - g(w-t)] \cot \tfrac{1}{2}t\,dt. \tag{2.13}$$

Hence, since $BT = \sum_1^\infty \alpha_n \sin nw$, we can write finally

$$u^* - 1 = A(u-1) - \tfrac{1}{2}(1-A)s + E\mathscr{H}(BT). \tag{2.14}$$

Notice that the factor A is the attenuation of an upstream flow variation by a uniform gauze normal to the stream, and that the remaining terms represent the disturbances introduced respectively by the variations of resistance coefficient and by variations of inclination to the stream. The separation of these effects is a consequence of the linearization of the equations. Equation (2.14) is a linear relation between the upstream velocity distribution, u, the downstream distribution, u^*, and the two properties of the gauze, and it may be used to relate any one of these quantities to the other three. For example, if the velocity distributions u and u^* are known, then there are two distinct cases of interest, corresponding to a shaped gauze of uniform resistance or a gauze of non-uniform resistance placed normal to the stream.

If, first, the distribution of inclination, T, is required explicitly, it is readily obtained from (2.14) by applying the transformation $\mathscr{H}^*$ to give

$$EBT = \mathscr{H}^*\left[\frac{u^*-u}{E} + (2-B)(u-1) + \tfrac{1}{2}(1-A)s\right]. \tag{2.15}$$

Provided u, u^*, K and B are given, (2.15) can be integrated to give the gauze shape, $x - x_0 = \int_{y_0}^{y} T(y)\,dy$, required to produce a prescribed velocity distribution.

Secondly, for a gauze placed normal to the stream so that $T = 0$, (2.14) gives the variation of resistance required to produce a prescribed velocity distribution as

$$s = 2[A(u-1) - (u^*-1)]/(1-A). \tag{2.16}$$

If this variation of s is produced by the variable spacing of a row of cylinders for which $\beta = 1 - d/l$, then by (1.5)

$$K \doteqdot \left(\frac{d}{l}\right)^2 \left(1 - \frac{d}{l}\right)^{-2} = K_1(1+s). \tag{2.17}$$

It is assumed that K_1 is given; hence (2.16) and (2.17) can be solved for d/l. A restriction is imposed by the requirement $d/l > 0$, so that

$$(u^*-1) < \tfrac{1}{2}(1-A)+A(u-1).$$

In particular, to produce the linear gradient $u^*-1 = \lambda(y/L-\tfrac{1}{2})$ from a uniform, incident flow with $u = 1$ requires

$$s = -2\lambda(y/L-\tfrac{1}{2})/(1-A). \tag{2.18}$$

This particular result has already been derived and verified experimentally by Owen & Zienkiewicz (1957).

3. Axisymmetric and diffuser flow

The simplest extension of the above results is to nearly radial, two-dimensional flow. Use polar co-ordinates (r,θ). For potential flow in a channel with walls at $\theta = 0, \alpha$, the velocity, distant from the gauze, is radial and of the form $u = f(\theta)/r$. The gauze produces a perturbation ψ^* to the stream function ψ^0, where again $\nabla^2\psi^* = 0$. Appropriate eigenfunctions for ψ^* are $r^k \sin k\theta$, where $k = n\pi/\alpha$. The solution proceeds exactly as in section 2, except that y, u_1, u_2 are replaced by $\theta, f_1(\theta), f_2(\theta)$. Equation (2.14) becomes

$$f^*-1 = A(f-1)+E\mathscr{H}(BT)-\tfrac{1}{2}(1-A)s, \tag{3.1}$$

where A, E, s, and the operator $\mathscr{H}$ have the same meaning as in § 2, but T is the tangent of the angle between the radius vector and the gauze normal. The solution (2.15) also applies, but $x-x_0$ is replaced by $\log r/r_0$, so that

$$\log\frac{r}{r_0} = \frac{\alpha}{\pi}\int_{\theta_0}^{\theta}\frac{1}{B}\mathscr{H}^*[(f^*-f)/E+(2-B)(f-1)]\,dw. \tag{3.2}$$

A further extension can be made to the axisymmetric flow in a circular pipe of radius a. Use cylindrical polar co-ordinates (r, θ, z), where $\partial/\partial\theta \equiv 0$ and $\mathbf{U} = (V, 0, U)$. The equation of continuity is satisfied by introducing a stream function defined by $rU = -\partial\psi/\partial r$, $rV = \partial\psi/\partial z$. As before, we assume that vorticity is conserved along a streamline, except for a discontinuity introduced at the gauze. Hence, the vorticity $\boldsymbol{\zeta}$ is of the form $(0, \zeta, 0)$, where

$$r\zeta = \frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2} = L(\psi), \text{ say.} \tag{3.3}$$

If the gauze produces a perturbation ψ^* to the stream function ψ^0, then $L(\psi^*) = 0$. This equation can be separated to yield the finite solutions $\psi^* = re^{-m|z|}J_1(mr)$, where $J_1(ma) = 0$. The solution now proceeds as before except that $J_0(mr)$ replaces $\cos mw$, $J_1(mr)$ replaces $\sin nw$, and the summations are over all roots m such that $J_1(ma) = 0$. The previous solution (2.16) applies except that the transformation $\mathscr{H}$ is now defined in terms of the Bessel functions J_0 and J_1 by

$$\left.\begin{aligned} g(r) = \sum_m h_m J_1(mr), \quad g^*(r) = \sum_m h_m J_0(mr), \quad J_1(ma) = 0 \\ \text{and} \qquad g^* = \mathscr{H}(g), \quad g = \mathscr{H}^*(g^*). \end{aligned}\right\} \tag{3.4}$$

The properties of the transformation represented by these Fourier–Bessel series can be expected to be similar to the corresponding Fourier series.

The solution for the axisymmetric flow in a circular pipe can be extended to the case of flow in a circular-cone diffuser, in a manner similar to that used for radial two-dimensional flow. Use spherical polar co-ordinates (r, θ, ϕ), so that $\partial/\partial\phi = 0$ and $\mathbf{U} = (U, V, 0)$. For potential flow within the cone $\theta = \alpha$, the velocity, distant from the gauze, is radial and of the form $u = f(\theta)/r^2$. As before, solution (2.16) with the modification (3.2) applies here, except that $f(\theta)$ replaces $u(y)$.

4. Multiple gauzes

A further simple extension of the previous results is to the common practical case of two or more gauzes placed in series. Consider a gauze A placed at $x = 0$ and a gauze B placed at $x = x_0$ and define

$$\eta_n = \coth \xi_n, \quad \zeta_n = \operatorname{coshec} \xi_n,$$

where $\xi_n = n\pi x_0/L$ and n is an integer. As before, it is possible to express the velocity field in terms of trigonometric series so that with, in an obvious notation,

$$\begin{aligned} u-1 &= \sum_{n=1}^{\infty} H_n \cos nw, \\ B_A T_A &= \sum_{n=1}^{\infty} G_n \sin nw, \\ u^*-1 &= \sum_{n=1}^{\infty} L_n \cos nw, \\ B_B T_B &= \sum_{n=1}^{\infty} M_n \sin nw, \end{aligned}$$

then it can be shown that

$$D_n L_n = P_n H_n + Q_n G_n + R M_n,$$

where
$$\begin{aligned} D_n &= -\gamma_A \gamma_B \zeta_n + [(1+\gamma_A)(1+\gamma_B) + (1-B_A)(1-B_B)]\eta_n \\ &\qquad + (1+\gamma_A)(1-B_B) + (1+\gamma_B)(1-B_A), \\ P_n &= -\gamma_A \gamma_B (1-B_A)(1-B_B)\zeta_n + [1 + (1-\gamma_A)(1-\gamma_B)(1-B_A) \\ &\qquad \times (1-B_B)]\eta_n + (1-\gamma_A)(1-B_A) + (1-\gamma_B)(1-B_B), \\ Q_n &= \gamma_A + (1-B_B)[\gamma_B(1+\gamma_A)\zeta_n + \gamma_A(1-\gamma_B)\eta_n], \\ R_n &= \gamma_B(1-B_A) + [\gamma_A(1-\gamma_B)\zeta_n + \gamma_B(1+\gamma_A)\eta_n]. \end{aligned}$$

For ξ_1 large, it can be shown that the solution corresponds to the result that would be obtained by assuming the gauzes did not interfere and using (2.16) twice. In general, although P_n can be zero, due to the presence of ζ_n and η_n in each coefficient, two interfering gauzes cannot remove all variations in $u-1$ but can merely remove a particular harmonic component. The case of two normal interfering gauzes has already been solved by Davis (1957).

5. The linear gauze

A linear gauze, inclined at a uniform angle to the stream, provides both theoretically and experimentally one of the simplest flows through a gauze inclined to the stream and at the same time reveals all the consequences of the ability of a gauze to deflect the stream. Consider the uniform flow incident on a gauze for which both T and K are uniform over the gauze surface and, therefore, s is zero. By (2.14),

$$u^* - 1 = EBT\mathscr{H}(1).$$

The transform $\mathscr{H}(1)$ can be evaluated by elementary analysis, giving

$$u^* - 1 = \frac{2EBT}{\pi} \log \cot \tfrac{1}{2}w. \tag{5.1}$$

In figure 2, the curve of $(u^* - 1)/EBT$ as a function of y is drawn together with experimental values. The agreement is good even close to the walls where the present theory gives an infinite value.

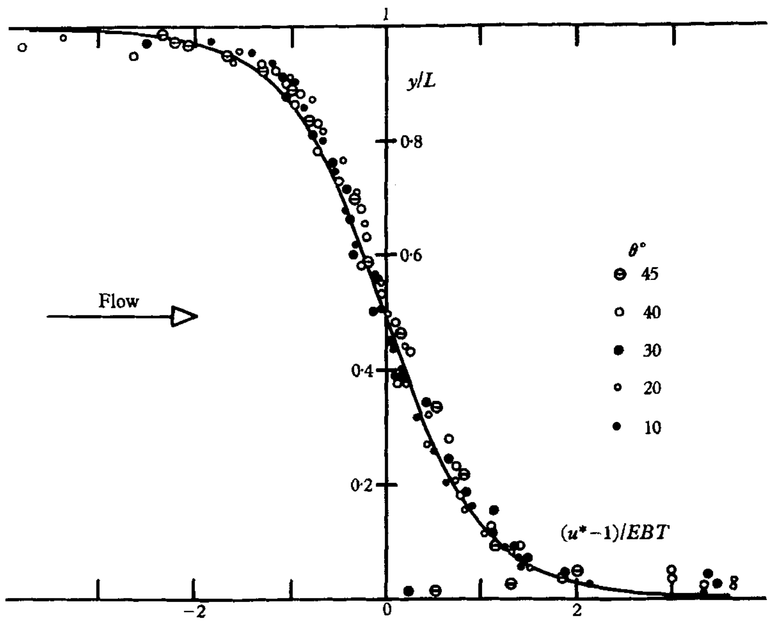

FIGURE 2. The velocity profile downstream of a gauze inclined at a uniform angle to a uniform incident stream. Theoretical curve from equation (5.1). Experimental points for $\theta = 10°, 20°, 30°, 40°, 45°$.

Apart from some qualitative observations in a water channel, the experiments were performed, in collaboration with Dr G. Davis of the Australian Atomic Energy Commission, in the wind tunnel of the Heat Research Laboratory of the Engineering Laboratory, Cambridge. The tunnel provided a velocity of 30 m/sec in a polished wood-working section of 5 in. × 10 in. Velocity measurements were made with a 0·125 in., 3-hole probe and an inclined tube manometer. The tunnel boundary layer was 0·05 in. thick. Velocity variations across the section, outside the boundary layers, was less than $\pm 1\%$ and flow direction variations were less than $\pm 15'$. The screens were mounted on wooden frames and clamped to the adjustable tunnel walls which provided a range of θ up to 45°. The data of figure 1 were with values of l, β, K, B of 0·0194 in., 0·395, 2·20 and 0·220, while the values for figure 4 were 0·0117 in., 0·348, 3·20 and 0·286.

Notice that the coefficient

$$ET = \sin 2\theta/(M+\cos 2\theta), \tag{5.2}$$

where

$$M = 1+(4-2B)/K,$$

has a maximum at $\cos 2\theta = -1/M$, where its value is $(M^2-1)^{-\frac{1}{2}}$. ET is plotted in figure 3, for a series of values of K and corresponding values of B taken from (1.6). It is seen that near $\theta = 0$ there is an extensive region where $ET \propto \theta$, that the extent of this region increases with K, and that the maximum occurs for $\theta \geqslant \frac{1}{4}\pi$. Although the maximum values of ET suggested here occur at values of

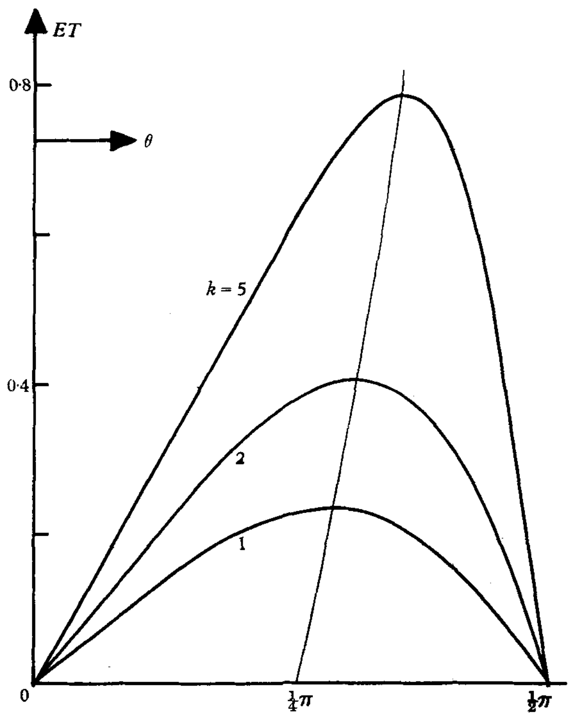

FIGURE 3. The deflexion parameter ET for various values of K as a function of the gauze inclination.

the inclination outside the range of the linearized solution, qualitative experiments in a water channel, in which we observe the deflexion of a dye stream on passing through the gauze, clearly show a maximum deflexion for $\frac{1}{4}\pi < \theta < \frac{1}{2}\pi$. Since the deflexion is proportional to ET, these observations suggest that the qualitative conclusions of the linearized theory apply even for $\theta > \frac{1}{4}\pi$.

It is important to evaluate the stream function in order to verify directly the fundamental assumption that the deflexion of a streamline in passing through the gauze is small. The deflexion $Y(x,y)$ of a streamline which passes through y and originates at y_∞ is

$$Y \equiv y - y_\infty,$$

and is related to the perturbation stream-function, to the first order, by

$$Y/L = \psi^*/V.$$

From (2.3) and (2.11) we obtain

$$\psi^* = [4EBT/\gamma_0\pi^2]\,S(x,y) \quad (x < 0)$$
$$= -[4EBT(1+\gamma_0)/\gamma_0\pi^2]\,S(x,y) \quad (x > 0),$$

where
$$S = \sum_{n\,\mathrm{odd}}^{\infty} \frac{1}{n^2} e^{-n|x|/L} \sin nw.$$

The form of the mid-stream line, for which $y_\infty = \frac{1}{2}L$, has been computed and is shown in figure 4. The bulk of the change occurs in $|x| < L$. Near $y = \frac{1}{2}L$ the final displacement
$$Y(\infty, y)/L \doteqdot EBT[-0{\cdot}371 + (y/L - \tfrac{1}{2})^2].$$

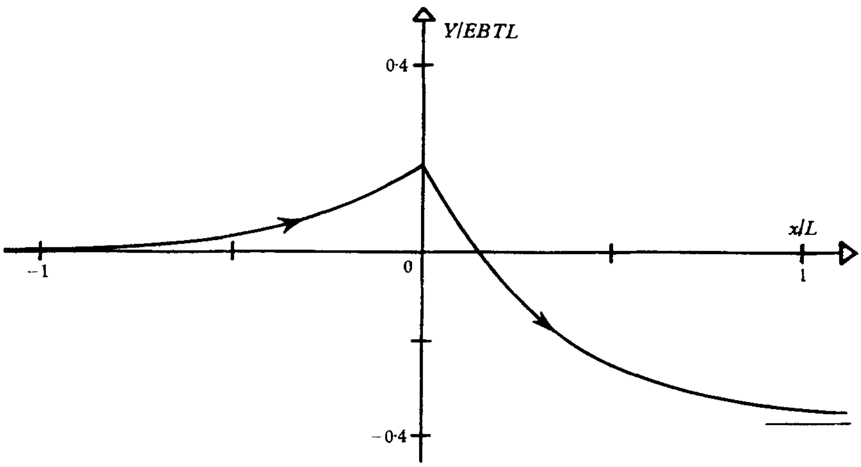

FIGURE 4. The middle streamline through a linear gauze placed at $x = 0$.

It is also of interest to observe that the stress coefficient is

$$p \equiv L\left\{\frac{\partial u^*}{\partial y}\right\}_{y=\frac{1}{2}L} = -2EBT,$$

and that the terms neglected in deriving the boundary conditions at the gauze are of order
$$[(V_1 - V_2)\,T]_{y=\frac{1}{2}L} = -EBT^2.$$

The experiments here were with values of p up to 0·2, whereas in the measurements of Owen & Zienkiewicz (1957) the stress reached 0·45. It thus appears that the linearized equations may be used with confidence up to $p = 0{\cdot}5$ and $T = 1$.

6. The parabolic gauze

A second simple flow through a non-normal gauze is that through a parabolic gauze. This introduces the additional feature that $K\cos^2\theta$ is no longer independent of y. Consider the gauze $(y - \frac{1}{2}L)^2 = L(kL - x)/4k$, of sag k, so that $T = dx/dy = -8k(w - \frac{1}{2}\pi)/\pi$. Hence by (2.14), with $u = 1$, $K =$ constant, but $s \neq 0$, and evaluating the transform $\mathscr{H}(w - \frac{1}{2}\pi)$ by elementary analysis,

$$(u^* - 1) - \Delta u = -\frac{8kEB}{\pi}\log 2\sin w, \tag{6.1}$$

where
$$\Delta u = \tfrac{1}{2}(1 - A)(1 - 4k\cos^2\theta/\tan^{-1} k).$$

In figure 5, $(u^* - 1 - \Delta u)/kEB$ is plotted together with experimental values. The agreement is not quite so good as for the linear screen, due to the difficulty of bending an accurate parabola.

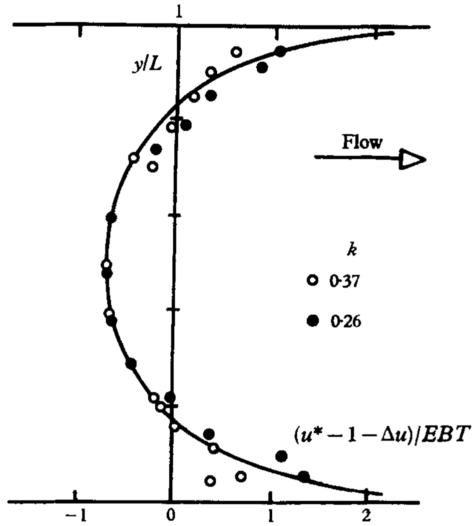

FIGURE 5. The velocity profile downstream of a parabolic gauze in a uniform stream. Theoretical curve from equation (6.1). Experimental points for sag $k = 0{\cdot}26$ and $0{\cdot}37$.

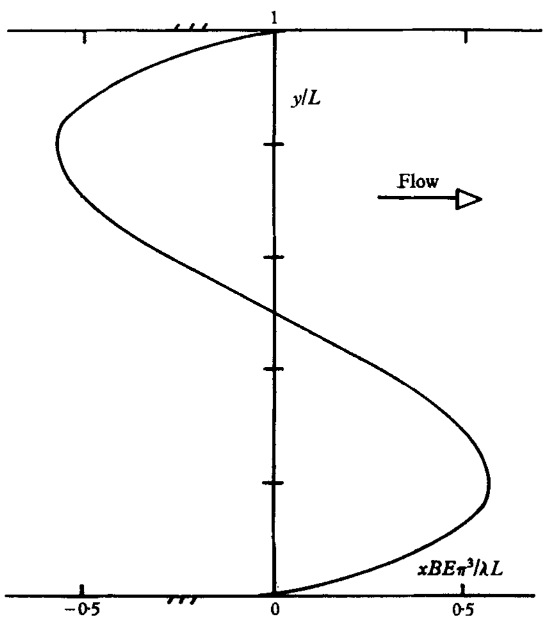

FIGURE 6. The gauze shape to produce a uniform shear from a uniform incident stream.

7. The production of a uniform shear

The production of a linear velocity gradient by means of a linear variation of resistance across the gauze as in (2.18) has already been investigated by Owen & Zienkiewicz (1957). The alternative physical method of bending a uniform gauze to an appropriate shape has some practical convenience. Consider the particular case of (2.15) required to produce the linear profile $u^* - 1 = \lambda(y/L - \frac{1}{2})$ from $u = 1$; it is

$$x(y) = \frac{\lambda L}{BE\pi^2}\int_0^a \mathscr{H}^*(a)\,da \quad (a = w - \tfrac{1}{2}\pi). \tag{7.1}$$

It can be shown by elementary analysis that

$$\begin{aligned}\mathscr{H}^*(w-\tfrac{1}{2}\pi) &= \frac{2}{\pi}\int_0^w \log\tan\tfrac{1}{2}t\,dt,\\ &= \frac{1}{\pi}(-C + a^2 + \tfrac{1}{12}a^4 + \tfrac{1}{240}a^6 + \ldots),\end{aligned} \tag{7.2}$$

where
$$C = -\frac{1}{2}\int_0^{\frac{1}{2}\pi} \log\tan\tfrac{1}{2}t\,dt = 0{\cdot}915,$$

so that
$$BE\pi^3 x/\lambda L = -0{\cdot}915a + \tfrac{1}{3}a^3 + \tfrac{1}{60}a^5 + \tfrac{1}{1680}a^7 + \ldots. \tag{7.3}$$

This series can be used with confidence right up to the wall since x becomes small there. The required gauze shape, given by $BE\pi^3x/\lambda L$ as a function of y, is shown in figure 6. This shape is to be expected in view of the result for a linear screen where in the centre of the channel the shear is uniform and only the flow near the walls would need to be adjusted to produce a completely uniform flow.

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